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Fins with temperature dependent surface heat flux—II. Multi-boiling heat transfer

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Abstract—An analytical and experimental study is conducted for a fin when various types of boiling occur simultaneously at adjacent locations on its surface. The heat transfer coefficient is taken as a power function of wall superheat in each heat transfer mode. The temperature distribution as well as the variation of length in each section is calculated for different base temperatures. Hysteresis effects and the sudden change in total heat transfer rate are particularly noted. Experiments were carried out with 2.5 cm diameter copper rods with boiling liquids of water and isopropyl alcohol. The results from theory are compared with experimental data.

INTRODUCTION

TRANSFER of high-density heat fluxes is an effective means in providing reliable operation and reducing the overall dimensions of a number of engineering devices. The use of finned surfaces in boiling liquids has made it possible to design compact systems which dissipate heat flux more than several folds greater that at an isothermal surface. In this case, different boiling modes are capable of stable coexistence on a fin. This phenomenon was clearly seen in experiments [1].

Not many studies dealing with boiling heat transfer from a single fin are found in the literature. Most of them [2–4] are engrossed in the investigation of single boiling mode on an extended surface. This is due to the difficulty in obtaining the temperature distribution when a fin is subject to the boiling liquids with the character of high nonlinearity. However, in many applications, various heat transfer modes, such as convection, nucleate boiling, transition boiling, and film boiling may occur simultaneously on a long fin depending on the designed operating condition. The prediction of heat duty in this problem is attractive.

Lai and Hsu [5] presented a one-dimensional model to study the heat transfer characteristics of a fin partly cooled by nucleate boiling and partly by convection. Good agreement is observed between their results and previous experimental data [6]. Based on some assumptions, Dul'kin *et al.* [7] derived analytical expressions to determine the temperature distribution when many modes of boiling occurred simultaneously on a fin. Their theoretical results were verified with their own data using copper pin fins (1.4 cm $D \times 6.6$ cm L) boiling in saturated water and Freon-113. Later, considering the coexistence of three boiling modes on a fin, Petukhov *et al.* [8] carried out a similar work. Recently, Unal [9] has proposed a model to calculate the heat duty of a fin upon which film, transition, and nucleate boiling show up steadily by imposing an insulated boundary at the free end. However, the heat transfer from the free end is important in common practice of heat transfer engineering.

All the foregoing researches have paid more attention to heat transfer than temperature. Hsu and Graham [10] pointed out that, in boiling heat transfer, fins are used not only to extend the heat transfer surface, but to render a broader range of operation when the danger of burnout is present. Hence the critical temperature at which the heat flux varies drastically is particularly noted. The purpose of this paper is to present a model to calculate the lengths in each boiling section, and to determine the heat flux at a given base temperature. The basic feature of this model is that the heat transfer process is governed by power-law-type dependence on temperature superheat. The coexistence of all the boiling modes is considered and a comparison is made between the cases with and without insulation at the end. Besides, the results are compared with experimental data.

ANALYTICAL APPROACH

A schematic diagram of a uniform fin subject to many types of heat transfer is shown in Fig. 1. The fin is cooled through a heat transfer coefficient by the ambient liquid. The heat transfer coefficient, h, varies depending on ΔT, the temperature difference between the fin wall and the ambient liquid. The steady state one-dimensional heat conduction equation is written as

$$\frac{\mathrm{d}^2 T}{\mathrm{d}x^2} - \frac{P \cdot h(\Delta T)}{k \cdot A} \cdot \Delta T = 0. \tag{1}$$

If the heat transfer coefficient in each boiling mode

NOMENCLATURE

A	cross-section area of fin
a	defined in equation (2)
F	hypergeometric function
h	heat transfer coefficient
k	thermal conductivity of fin
L	length of fin
L_j	length of partitioned fin
m	power-law exponent, equation (2)
N	fin parameter, equation (4)
$N_{\max}$	maximum fin parameter in transition boiling
P	periphery of fin
Q	dimensionless heat flux
q	heat flux
T	temperature
X	dimensionless coordinate, Fig. 1.

Greek symbols

β	defined in equation (9)
ΔT	wall superheat
θ	dimensionless temperature.

Subscripts

b	fin base
j	a chosen partition of a fin
k	last section, Fig. 1
0	right boundary of j-section
1	left boundary of j-section.

can be approximated by a power-law-type formula on wall superheat, the expression in the j-section of the fin as shown in Fig. 1 is

$$h_j(\Delta T) = a_j \cdot \Delta T^{m_j - 1}. \tag{2}$$

In the above equation, m is a constant in each segment, but can vary from one segment to another.

Introducing the dimensionless quantities $X_j = x_j/L_j$ and $\theta_j = \Delta T_j/\Delta T_{j1}$, equation (1) becomes

$$\frac{d^2\theta_j}{dX_j^2} - N_j^2 \cdot \theta_j^m = 0 \tag{3}$$

where the fin parameter, N_j, is defined as

$$N_j = L_j \cdot \sqrt{\frac{Ph_{j1}}{kA}} \tag{4}$$

and subject to the boundary conditions

$$\theta_{j1} = 1 \tag{5}$$

$$\frac{d\theta_j}{dX_j}(0) = Q_{j0}. \tag{6}$$

The temperature distribution in the j-section can be obtained by integrating equation (3) and using boundary conditions (5) and (6). The detailed derivation can be found in the companion paper [11].

For $m > -1$

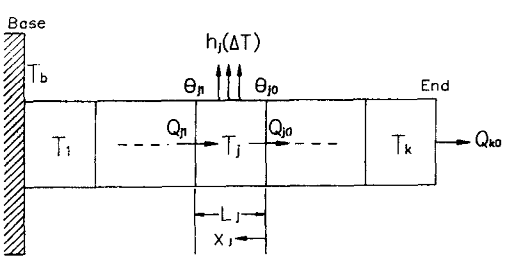

FIG. 1. Configuration of the proposed model.

$$N_j \cdot X_j = \sqrt{\left[\frac{2}{m_j+1} \cdot (\theta^{-m_j+1} - \theta^{-2m_j}\beta)\right]} \cdot F\left[1, \frac{m_j}{m_j+1}; \frac{3}{2}; 1-\beta \cdot \theta_j^{-m_j-1}\right] - Q_{j0} \cdot N_j^{-1} \cdot \theta_{j0}^{-m_j} \cdot F\left[1, \frac{m_j}{m_j+1}; \frac{3}{2}; 1-\beta \cdot \theta_{j0}^{-m_j-1}\right] \tag{7}$$

and for $m < -1$

$$N_j \cdot X_j = \theta_j \cdot \sqrt{\left[\frac{2}{m_j+1} \cdot (\theta^{m_j+1}\beta^{-2} - \beta^{-1})\right]} \cdot F\left[1, \frac{m_j+3}{2(m_j+1)}; \frac{3}{2}; 1-\beta^{-1}\theta_j^{m_j+1}\right] - Q_{j0} \cdot N_j^{-1} \cdot \theta_{j0} \cdot \beta^{-1} \cdot F\left[1, \frac{m_j+3}{2(m_j+1)}; \frac{3}{2}; 1-\beta^{-1}\theta_{j0}^{m_j+1}\right] \tag{8}$$

where F represents hypergeometric function and β is denoted as

$$\beta = \theta_{j0}^{m_j+1} - \frac{m_j+1}{2} \cdot N_j^{-2} \cdot Q_{j0}^2. \tag{9}$$

The heat transfer rate at the left boundary of j-section, Q_{j1}, is calculated as

$$Q_{j1} = \sqrt{\left(\frac{2}{m_j+1} \cdot N_j^2 \cdot (1-\theta_{j0}^{m_j+1}) + Q_{j0}^2\right)}. \tag{10}$$

The continuity equations of temperature and heat flux at the intersection of two elements are written as

$$(\Delta T_j)_0 = (\Delta T_{j+1})_1 \tag{11}$$

$$(Q_j \cdot \Delta T_j/L_j)_0 = (Q_{j+1} \cdot \Delta T_{j+1}/L_{j+1})_1. \tag{12}$$

At the end of the fin, two cases are considered respectively. Firstly, if the end is exposed to ambient liquid, then it is

$$Q_{k0} = \frac{L_k}{k} \cdot h_{k1} \cdot \theta_{k0}^{m_k}. \tag{13}$$

Otherwise, in case the end is insulated, it leads to the following boundary condition

$$Q_{k0} = 0. \tag{14}$$

EXPERIMENTS

Experimental apparatus

The apparatus used in this study is shown in Fig. 2. It was designed to allow simultaneous measurements of heat transfer and observations of the front and side views of the fin in boiling conditions. This apparatus consists mainly of a test section, a viewing chamber, and accessories to maintain the system in saturated boiling. The test section is mounted on one side of the viewing chamber, and the glass windows are placed on the remaining three sides for observations. Pure copper (99.99%) is used as the material to construct the test section. The cylindrical test fin with a diameter of 2.5 cm was machined from one end of a solid cylinder. The other end of the cylinder was connected tightly with a square copper block, which was drilled to fit twelve cartridge heaters, each rated at 500 W. The power supplied to the cartridge heaters is controlled by two sets of variable auto-transformers. Twelve chromel–alumel thermocouples (K-type, 30 gage) are positioned as shown in Fig. 3. Fiberglass blocks are used as the insulation material to achieve an isothermal condition at the fin base. In addition, the square heating block was wrapped with several layers of ceramic fiber to minimize the heat loss.

The test liquid was contained in a square duct of cross section 15 by 15 cm. An immersion heater located at the bottom of the chamber was used to maintain the liquid at its saturation temperature. The liquid temperature was determined by a thermocouple located at the center of the pool which is about 5 cm above the test piece. At the top of the chamber, cold water was made to pass through a coil to condense the vapor being generated.

Experimental procedure

The surface of the test fin was cleaned by commercial 'Brasso' liquid and rubbed by dry paper wipers until it was shining. The contact angle measured from the liquid side served as an indicator of the degree of surface wettability. A smaller contact angle accords with a more wettable surface. With a water droplet placed on a clean copper surface, the contact angle was found to be 90°. For the case of an insulated tip, a block of fiberglass was tightly pressed between the free end and the window. Various lengths of the fin are obtained by removing the end portion by milling.

Start-up of a typical experimental run began with the deaeration of the working fluid by vigorous boiling in a reservoir as well as preheating of the test section. During the preheating period, the chamber was filled with nitrogen gas to avoid oxidation of the test surface. Firstly, a heating experiment is conducted. As

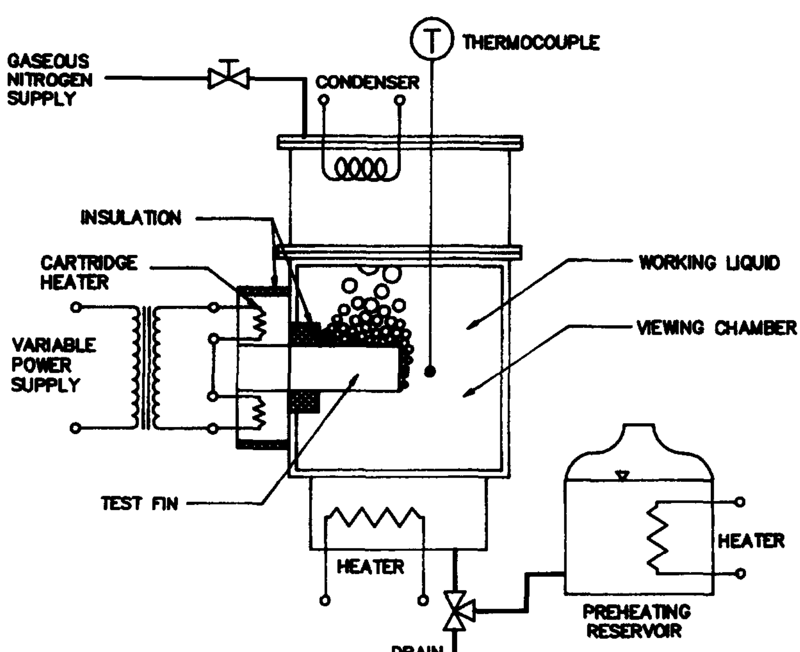

FIG. 2. Schematic diagram of the experimental apparatus.

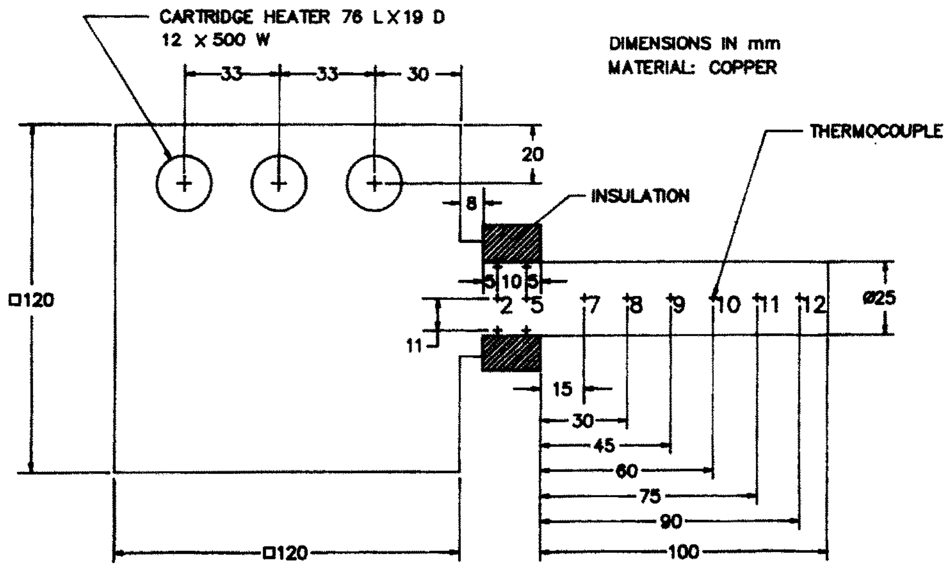

FIG. 3. Details of 10 cm test pin fin and locations of thermocouples.

the tip temperature of the fin was heated up to a temperature slightly above the saturation temperature of the liquid, the chamber was then filled with working fluid from the reservoir and the boiling process commenced. The base temperature of the test fin was increased by adjusting the voltage input of the cartridge heaters. Tests were considered to reach steady state if the drift of temperature readings was less than 1 K in five minutes. As the temperature of the test section reached steady state condition, all thermocouple outputs were recorded directly using a data recorder (Yokogawa model HR-2500). The power was increased step by step and the fin was heated until nucleate and transition boiling disappeared. The whole fin was then subject to film boiling and a heating run was completed. In a cooling run, the power of the heater was gradually reduced. As the base temperature of the fin decreased, steady state data were also recorded in the same way. The procedure was continued until the vapor film on the fin surface collapsed.

RESULTS AND DISCUSSION

In the theoretical approach, the fin is subdivided into many segments depending on the type of boiling. The heat transfer coefficient is taken from the usual pool boiling curve, namely h versus ΔT, obtained on a large flat test surface at a series of isothermal temperatures [12]. The temperature and heat flux between two adjacent sections are continuous. With the boundary condition of a prescribed temperature at the root and either equation (13) or (14) at the end of a fin, the governing equation is a well-posed elliptic differential equation.

The solution is obtained by an iterative shooting procedure. Initially, the tip temperature of the fin is guessed. Since the temperature at the junction of two segments is known, the length and the total heat transfer rate in the k-section can be calculated by the specified end condition of equation (13) or (14). Then the length of the neighboring segment is obtained by the same way consecutively. Subsequently, the next guess in tip temperature is based on the deviation of the total length and the actual fin length. It continues until the relative error is less than 10^{-4}.

A successful experiment has been performed by Bui [13]. Using saturated water as the boiling liquid, a complete boiling curve was acquired on a vertical copper surface (6.3 cm $W \times$ 10.3 cm H). In the case of a clean surface with a contact angle of 90°, the superheat at the incipience of nucleate boiling is 4.4 K, and the maximum and minimum wall superheats on the boiling curve are at 18.5 and 72 K, respectively. A set of approximate power-law functions of surface flux was obtained and listed in Table 1. In addition, the expression regarding isopropyl alcohol has been adopted from Unal [9] and the original data produced by Haley and Westwater [6]. The values of a and m were used as input during the calculation.

Both heating and cooling experiments were performed with saturated water and isopropyl alcohol at one atmosphere pressure. Experiments on test fins with and without insulation at their ends were carried out respectively. First of all, a 10 cm fin is tested when the system is initially at the saturation temperature of the boiling liquid. The measured temperature readings at the center line of the fin are plotted in Fig. 4. The temperature as well as the absolute value of its slope decreases along the fin axis. The base temperature was estimated from the thermocouples, 2, 5, and 7, while the base heat flux was determined on the basis of temperature gradient across the thermally insulated region. The typical boiling curve regarding the variations of base heat flux on ΔT_b is shown in Fig. 5. In this figure, both heating and cooling runs for $L = 8.5$ and 10 cm are included. At a given base temperature,

Table 1. Correlations of surface heat flux

$q = a \cdot \Delta T^m$ ($W\ m^{-2}$) Type	Water		Isopropyl alcohol	
	a	m	a	m
Film boiling	3100	0.5	254	1
Transition boiling	7.55×10^8	-2.46	4.7×10^7	-1.5
Nucleate boiling	157.6	2.81	28	3
Convection	1402	1.33	338	1.33

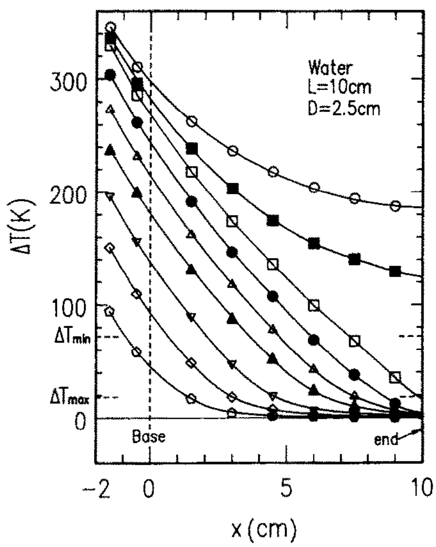

FIG. 4. Measured temperatures at the centerline of the 10 cm fin (heating).

the heat flux on the heating curve is much higher than that on the cooling one. For $L = 10$ cm, when ΔT_b is raised to 280 K, transition and nucleate boiling start to push off. The heat flux then decreases abruptly and the fin is completely blanketed with a vapor film. On the other hand, when ΔT_b is cooled to 90 K, transition and nucleate boiling show up at the fin tip and the total heat transfer rate increases quickly. Hence there are two critical temperatures at which heat flux jumps appear on a boiling curve, and a hysteresis region is formed. The critical temperature as well as the heat flux is lower for a shorter fin ($L = 8.5$ cm) as shown in this figure. It is interesting to note that the highest heat flux is about twice of that on a vertical isothermal surface [13], but the burnout temperature is shifted from 18.5 to 280 K with the addition of the fin. Therefore the advantage of using a fin in boiling liquids is not only to enhance heat transfer but to extensively broaden the range of safety operating temperature. Besides, a wide range of high heat transfer rate is more satisfactory for thermal control as compared with the boiling curve with a sharp peak on a flat surface.

The theoretical heat transfer rates are also depicted in Fig. 5. It is favorable to compare the results with experimental data. The heat flux as well as the critical temperature obtained from cooling is close to the data. However the predicted burnout temperature in a heating run is about 30 K higher than the measured value. It is suspected that the boiling curve on a flat plate is different from that on a rod. Based on 900 sets of data, the maximum heat flux was observed to be only 90% of that on a flat plate for large horizontal cylinders [14]. Therefore burnout may occur earlier in experiments.

Figure 6 presents temperature distributions when many types of boiling coexist simultaneously on an 8.5 cm fin in heating runs. With and without the tip insulated, the measured temperatures are compared with the predictions made from the present model by

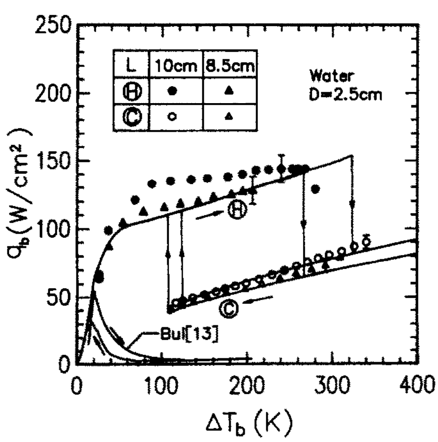

FIG. 5. Comparison of predicted and measured base heat fluxes. (Ⓗ : heating, © : cooling.)

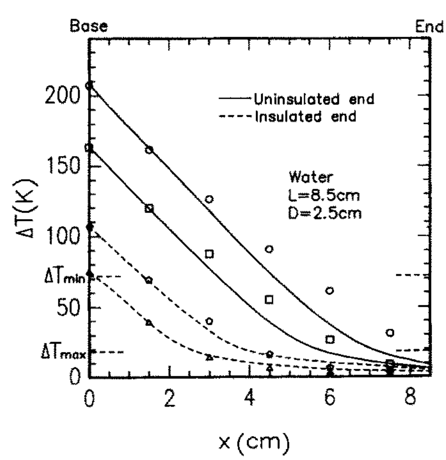

FIG. 6. Comparison of predicted and measured temperature profiles for a fin subject to multi-boiling modes.

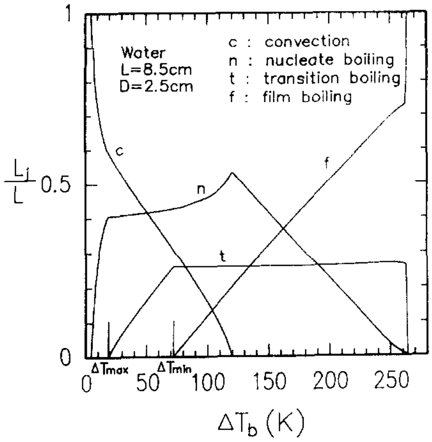

FIG. 7. Dependence of boiling length on ΔT_b in a heating run.

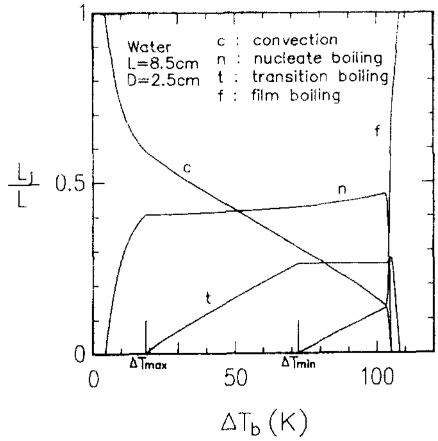

FIG. 8. Dependence of boiling length on ΔT_b in a cooling run.

using the same ΔT_b. Good agreement between them is observed. The boiling modes from root to end are film, transition, and nucleate boiling, respectively. The variation of ratio of length in each section is shown in Fig. 7. In a heating run, the region of nucleate boiling expands, accompanied by a shrinkage of the convective heat transfer region. As the base temperature increases further, transition boiling commences at the root of the fin, and the region of convection is gradually pushed to the end. Thereafter, film boiling shows up at the fin base and most of the fin is at highly efficient heat transfer modes of transition and nucleate boiling. The length over which transition boiling occurs is nearly constant, and the location moves toward the fin end as the base temperature increases. It is desirable to operate heat exchanger equipments in this condition. The increase in heat transfer rate is not pronounced with further increase in base temperature. Subsequently, nucleate boiling begins to push off and transition boiling disappears at the same time. At this point the whole fin is governed by film boiling at a poor heat transfer rate. The sudden decrease in the ability of heat removal may cause damage of thermal systems and is known as burnout. However the critical temperature is significantly raised as compared with that on an isothermal surface by 18.5 K.

The variation of the length ratio in a cooling run is shown in Fig. 8. When a fin is initially at very high temperatures, the heat is mainly transferred by conduction through a thin vapor film attached to the surface. As the temperature decreases, the layer of film becomes thinner. Eventually a little liquid-touch happens at the end portion of the fin. According to the results in the companion paper [11], transition boiling with a negative m can exist stably when N is less than N_{max}. As the transition boiling extends, the tip temperature decreases and the heat transfer from the end increases which results in the lowering of N_{max}. Therefore, the coexistence of film and transition boiling only appears in a small range of base temperature of about 3 K. However the length ratio of transition boiling can be as long as 27% of the entire fin before it becomes unstable. It is also seen in the experiments that the end portion in transition boiling exists steadily for at least five minutes. The photograph is displayed in Fig. 9 at $\Delta T_b = 98$ K. Thereafter, nucleate boiling and natural convection suddenly come in at the same moment and the heat flux is significantly raised. In the last stage, film and transition boiling move toward the fin base and eventually disappear. Natural convection becomes dominant and the total heat transfer rate drops.

A few experiments were also conducted with isopropyl alcohol. The dependence of base heat flux on ΔT_b for $L = 8.5$ cm with and without an insulated end is shown in Fig. 10. In general, the heat transfer rate is small when the end is insulated. With insulation applied to the end, the burnout temperature is significantly lowered, but no evident effect is detected in cooling runs. The results of the corresponding experiments are also plotted in this figure. The data are close to the prediction especially for the end-insulated one. It is interesting to compare the heat transfer rate

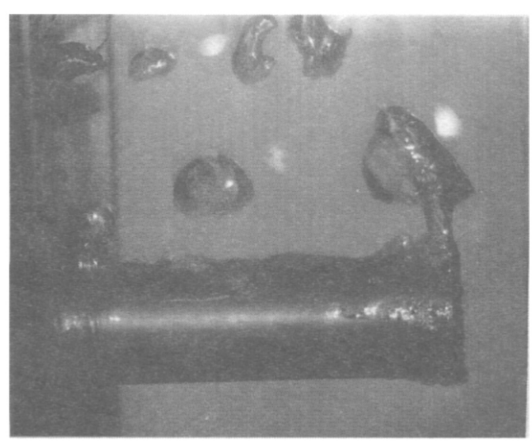

FIG. 9. Photograph of a fin with transition boiling at the end portion in a steady state. (Cooling, $\Delta T_b = 98$ K, $L = 8.5$ cm and $D = 2.5$ cm.)

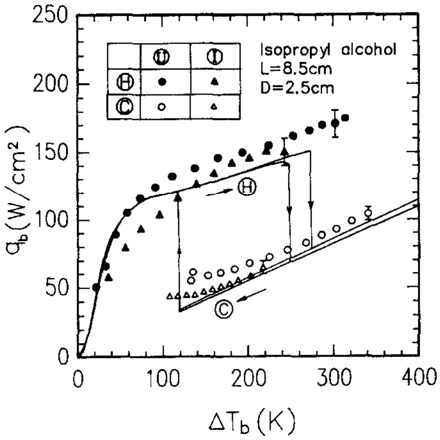

FIG. 10. Comparison of predicted and measured base heat fluxes in isopropyl alcohol. (Ⓤ : uninsulated end ; Ⓘ : insulated end.)

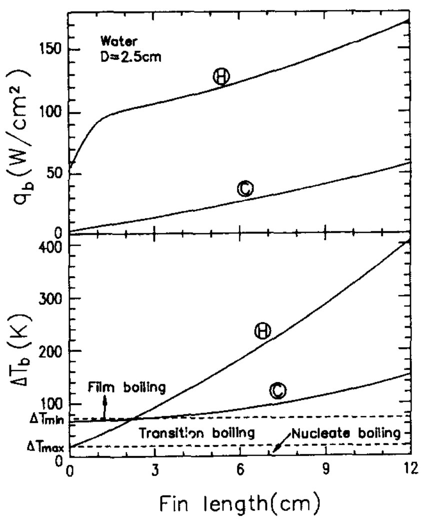

FIG. 11. Predictions of critical ΔT_b and q_b of fins referring to the local temperature and conductive heat flux of a 12 cm fin.

for different boiling liquids. On an isothermal flat plate, the maximum heat flux in isopropyl alcohol is only one-third of that in water, and a higher maximum heat transfer rate is observed in isopropyl alcohol in a long fin. This is due to a lower local heat transfer rate at the peripheral surface, and the region occupied by efficient nucleate and transition boiling is thus wider in boiling with isopropyl alcohol.

The knowledge of the critical temperature obtained in a cooling run is attractive because it is important in quenching materials to obtain a desired surface hardness which depends on the cooling rate. On the other hand, in a heating process, it provides a warning signal to operate heat exchangers within the temperature limit and free from burnout. Both of the critical temperatures were predicted by the present model. To find out the jumps, several iterative manipulations should be carried out in each ΔT_b. Now a simple way is proposed to predict them by referring to a longer fin. For example, a 12 cm fin has already been studied and the temperature profile on the occurrence of heat flux jump is shown in Fig. 11. As for other fins, it can be taken as another fin partitioned from the end part of the 12 cm fin. From the reading of Fig. 11, the critical temperatures for 8.5 and 10 cm fins are 260 and 320 K in heating runs and 110 and 122 K in cooling runs, respectively. It is exactly the same as the theoretical prediction given in Fig. 5. It should be pointed out that the proposed method is valid only when the base temperature of the fin is higher than ΔT_{min} so that the multi-boiling mode is the same as the referred fin.

The conductive heat flux inside the 12 cm fin is shown in the upper part of Fig. 11 as a function of the axial distance starting from the fin end. In a similar manner, for a fin other than 12 cm, the maximum and minimum heat transfer rates can be found in this figure. From the results of Fig. 11, all the heat fluxes and the critical temperatures increase with fin length at a chosen diameter. In addition, the deviation between two critical temperatures becomes large, which implies that the hysteresis effect becomes significant for a long fin.

CONCLUSIONS

The coexistence of multi-type boiling on a fin has been studied both analytically and experimentally in this work. The results are summarized as follows.

(1) The temperature distribution as well as the base heat flux is predicted by the present model.

(2) The length in each boiling mode is calculated, and its variation is noted in accordance with the change in base heat flux.

(3) Experiments are conducted for fins with and without end insulated in boiling liquids of water and isopropyl alcohol. The results compare favorably with theoretical predictions.

(4) The burnout temperature is significantly raised with the installation of a fin on the heater surface.

(5) Jumps in heat flux are observed in both heating and cooling runs. A simple method is proposed which predicts successfully the temperatures and their corresponding heat flux when jump occurs by referring to a long fin.

(6) A hysteresis region appears on the boiling curve of a fin. This hysteresis effect becomes significant in a long fin.

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